

Universal descriptions of chemical freeze-out based on pressure and specific heat respectively

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Abstract

Two new universal descriptions of chemical freeze-out parameters have been introduced based on pressure and specific heat respectively calculated in hadron resonance gas (HRG) model. It is shown that the chemical freeze-out parameters obtained at various $\sqrt{s_{NN}}$ approximately correspond to $P/T^4 = 0.88$ and $C_V/T^3 = 47$ respectively.

Keywords: Heavy Ion collision, Hadron Resonance Gas model, Chemical Freeze-out

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1. Introduction

In the last few years, a substantial amount of experimental and theoretical efforts have been devoted worldwide to investigate the strongly interacting matter under extreme conditions of temperature and/or baryon chemical potential. While at low baryon chemical potential and high temperature lattice quantum chromodynamics (LQCD) data seem to indicate a smooth crossover from hadronic to quark-gluon plasma (QGP) phase [1], at high baryon chemical potential and low temperature the system is expected to have a first-order phase transition [2]. This first order phase transition line at high baryon chemical potential and low temperature should end at a critical end point (CEP), a second-order phase transition point as one moves towards the high temperature and low baryon chemical potential region, in the QCD phase diagram. Heavy ion collisions provide a unique tool to create and study strongly interacting matter under extreme conditions of temperature and/or baryon chemical potential.

One of the primary goals of heavy-ion collision experiments is to map QCD phase diagram in terms of temperature and baryon chemical potential. At present, the properties of QCD matter at high temperature and almost zero baryon chemical potential are being investigated using ultra relativistic heavy ion collisions at the Large Hadron Collider (LHC), CERN and Relativistic Heavy Ion Collider (RHIC), Brookhaven National Laboratory (BNL). The Beam Energy Scan (BES) program of RHIC is currently investigating the location of the CEP. The HADES experiment at GSI, Darmstadt is investigating medium of very large baryon chemical potential. The region of large baryon chemical potential will also be explored by the NA61-SHINE experiment at CERN-SPS. In future, the Compressed Baryonic Matter (CBM) experiment at the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-based Ion Collider fAcility (NICA) at JINR, Dubna will also study nuclear matter at large baryon chemical potential.

LQCD provides the most direct approach to study the QCD matter at high temperature. However, at finite chemical potential, LQCD faces the well-known sign prob-

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lem. On the other hand, the hadron resonance gas (HRG) model [3–34], which is used in this present work, provide a simpler alternative to study strongly interacting matter at finite temperature and chemical potential. The HRG model is quite successful in describing the bulk properties of hadronic matter in thermal and chemical equilibrium [13, 14, 19]. This model is also successful in describing the ratios of hadron yields, at chemical freeze-out, created in central heavy ion collisions from SIS up to RHIC energies [6, 7, 9–11, 15–17]. In a heavy ion collision experiment, the chemical freeze-out is defined as the stage in the evolution of the thermal system when inelastic collisions among the particles cease and the particle ratios become fixed. At various center-of-mass energies $\sqrt{s_{NN}}$, particle yields or ratios are generally analysed phenomenologically using HRG model [15, 16, 35–39] to determine the chemical freeze-out parameters. Relations of chemical freeze-out temperature and baryon chemical potential with $\sqrt{s_{NN}}$ establish the chemical freeze-out line [16, 35] in the QCD phase diagram. The main result of these investigations is that the chemical freeze-out temperature increases sharply from SIS up to SPS energies and reaches, for higher collision energies, at constant values near $T = 160 - 165$ MeV while baryon chemical potential decreases sharply as a function of $\sqrt{s_{NN}}$. Chemical freeze-out is the earliest stage in the evolution of the hadronic phase which can be determined phenomenologically from the experiment data. Therefore, the chemical freeze-out line is very much important in the QCD phase diagram. There are several universal chemical freeze-out descriptions in the existing literature which can approximately describe the chemical freeze-out line in the QCD phase diagram. Those universal chemical freeze-out descriptions are independent of $\sqrt{s_{NN}}$ and they are related to the bulk thermodynamic properties of the system. Finding out the universal conditions of chemical freeze-out parameters have been the subject of various studies.

Our aim in the present work is twofold. First, we would like to study some basic bulk thermodynamic properties like pressure, energy density and specific heat of the matter in the HRG and an excluded volume HRG model. Second, we would like to find out universal conditions of chemical freeze-out descriptions from those thermodynamic quantities.

The paper is organised as follows. First, the ideal HRG model and an excluded volume HRG model are

briefly discussed. Then the region of applicability of non-interacting HRG model and its interacting version i.e., EVHRG model have been discussed using the LQCD data of pressure and energy density. Thereafter, results of specific heat have been presented. Finally, two new universal chemical freeze-out descriptions based on pressure and specific heat respectively have been proposed.

2. Hadron Resonance Gas model

In the HRG model, the thermal system consists of all the hadrons and resonances. There are varieties of HRG models in the existing literature. Different versions of this model and some of the recent works using these models may be found in Refs. [3–34]. The partition function of a hadron resonance gas in the grand canonical ensemble can be written as

$$\ln Z^{id} = \sum_i \ln Z_i^{id}, \quad (1)$$

where the sum is over all the particles. *id* refers to ideal i.e., non-interacting HRG model. For particle species *i*,

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \quad (2)$$

where *V* is the volume of the thermal system, *T* is the temperature, *g_i* is the degeneracy factor, *m_i* is the mass, $E_i(p) = \sqrt{p^2 + m_i^2}$ is the single particle energy and $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ is the chemical potential. In the last expression, *B_i*, *S_i*, *Q_i* are respectively the baryon number, strangeness and electric charge of the particle, μ 's are corresponding chemical potentials. The upper and lower sign corresponds to baryons and mesons respectively. In this present work, all the hadrons and resonances listed in the particle data book up to a mass of 3 GeV [40] have been incorporated. It is assumed that the hadronic matter is in thermal and chemical equilibrium. Therefore non-equilibrium phenomena like decays of particles have been ignored in this present work. The partition function is the basic quantity from which one can calculate various thermodynamic quantities of the thermal system. The pressure P^{id} , the energy density ε^{id} of the thermal system and the number density n_i^{id} of *i*-th particle can be calculated using the standard definitions,

$$\begin{aligned}
P^{id} &= \sum_i P_i^{id} = \sum_i T \frac{\partial \ln Z_i^{id}}{\partial V} \\
&= \sum_i \pm \frac{g_i T}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)],
\end{aligned} \tag{3}$$

$$\begin{aligned}
\varepsilon^{id} &= \sum_i \varepsilon_i^{id} = - \sum_i \frac{1}{V} \left(\frac{\partial \ln Z_i^{id}}{\partial \frac{1}{T}} \right)_{\frac{\mu}{T}} \\
&= \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i,
\end{aligned} \tag{4}$$

$$\begin{aligned}
n_i^{id} &= \frac{T}{V} \left(\frac{\partial \ln Z_i^{id}}{\partial \mu_i} \right)_{V,T} \\
&= \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}.
\end{aligned} \tag{5}$$

In case of heavy-ion collision experiments, the parameters T and μ 's of HRG model corresponds to those at chemical freeze-out which are believed to depend on initial conditions of the collision. The chemical potentials μ_B, μ_S and μ_Q are not independent but related (on average) to each other as well as to the T via the relations [41]

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q) S_i = 0, \tag{6}$$

and

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q) Q_i = r \sum_i n_i(T, \mu_B, \mu_S, \mu_Q) B_i, \tag{7}$$

where r is the ratio of net-charge to net-baryon number of the colliding nuclei. For central Au + Au or Pb + Pb collisions $r = N_p/(N_p + N_n) = 0.4$, where N_p and N_n are respectively proton numbers and neutron numbers of the colliding nuclei.

3. Excluded volume hadron resonance gas model

In the ideal HRG model particles are point-like. Although attractive interactions between hadrons are incorporated through the presence of resonances, repulsive

interactions are ignored completely in this framework. However, the repulsive interactions are also needed, especially at very high temperature and/ or large baryonic chemical potential, to catch the basic qualitative features of strong interactions where the ideal gas assumption becomes inadequate. In the EVHRG model [3–5, 8, 18, 19, 21, 22, 24, 26–29, 42, 43], hadronic phase is modelled by a gas of interacting particles, where the geometrical sizes of the particles are explicitly incorporated as the excluded volume correction to approximate the short-range van der Waals type repulsive interaction.

In EVHRG model pressure can be written as

$$P(T, \mu_1, \mu_2, \dots) = \sum_i P_i^{id}(T, \hat{\mu}_1, \hat{\mu}_2, \dots), \tag{8}$$

where for the i -th particle the effective chemical potential is

$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, \dots) \tag{9}$$

where $V_{ev,i} = 4 \frac{4}{3} \pi R_i^3$ is the volume excluded for that particle with hardcore radius R_i . In an iterative procedure, one can get the pressure. The pressure $P(T, \mu_1, \mu_2, \dots)$ is suppressed compared to the ideal gas pressure P^{id} because of the smaller value of effective chemical potential. The other thermodynamic quantities like ε , n_i can be calculated from Eqs. 8 - 9 as

$$\varepsilon = \varepsilon(T, \mu_1, \mu_2, \dots) = \frac{\sum_i \varepsilon_i^{id}(T, \hat{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}, \tag{10}$$

$$n_i = n_i(T, \mu_1, \mu_2, \dots) = \frac{n_i^{id}(T, \hat{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}. \tag{11}$$

This EVHRG model is thermodynamically consistent. Recently, effects of excluded-volume have been studied in the equation of state of pure Yang-Mills theory [44].

The difference of the EVHRG model, as compared to HRG, is governed by the radius parameter. The value of the hardcore radius was estimated as $R_i = 0.3$ fm in the Ref [9]. Here in this present work continuum limit LQCD data [45] of normalised pressure and energy density calculated at $\mu = 0$ have been used to understand the effect of radii of particles in EVHRG model. Using those LQCD

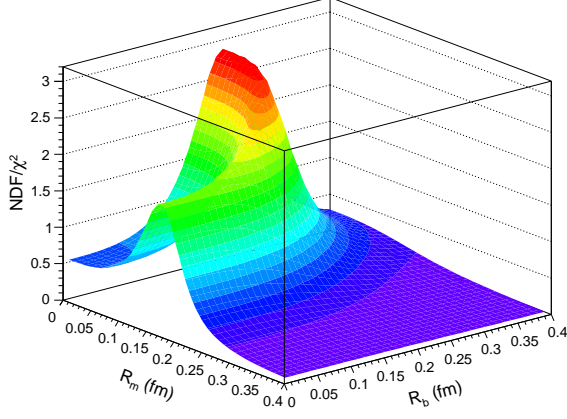


Figure 1: (Color online). Variation of inverse of χ^2/NDF with the hardcore radii of mesons (R_m) and baryons (R_b) at $\mu = 0$. LQCD data [45] up to $T = 200$ MeV have been used to calculate χ^2/NDF . Peaks in this surface plot correspond to the minima of χ^2/NDF .

data, χ^2 has been calculated at different radii of mesons R_m and baryons R_b where χ^2 has been defined as

$$\chi^2 = \sum_T \left[\frac{\left(\frac{P}{T^4}^{LQCD} - \frac{P}{T^4}^{EVHRG}(R_m, R_b) \right)^2}{\left(\Delta \frac{P}{T^4}^{LQCD} \right)^2} + \frac{\left(\frac{\varepsilon}{T^4}^{LQCD} - \frac{\varepsilon}{T^4}^{EVHRG}(R_m, R_b) \right)^2}{\left(\Delta \frac{\varepsilon}{T^4}^{LQCD} \right)^2} \right]. \quad (12)$$

In the last expression, $\Delta \frac{P}{T^4}^{LQCD}$ and $\Delta \frac{\varepsilon}{T^4}^{LQCD}$ are the errors of normalised pressure and energy density respectively calculated in the LQCD.

Figure 1 shows variation of inverse of the χ^2/NDF with R_m and R_b where NDF corresponds to the number of degrees of freedom. The upper limit of temperature for the continuum limit LQCD data [45] is taken as $T = 200$ MeV to calculate χ^2/NDF of Fig. 1. Peaks in this surface plot correspond to the minima of χ^2/NDF i.e., the best fit of the LQCD data. It can be seen from this plot that NDF/χ^2 is maximum for $R_m = 0.0$ fm and $R_b = 0.25$ fm.

Figure 2 is same as Fig. 1 but for the continuum limit LQCD data [45] up to $T = 170$ MeV which is close to the crossover temperature T_c (154 ± 9 MeV) [45] of the

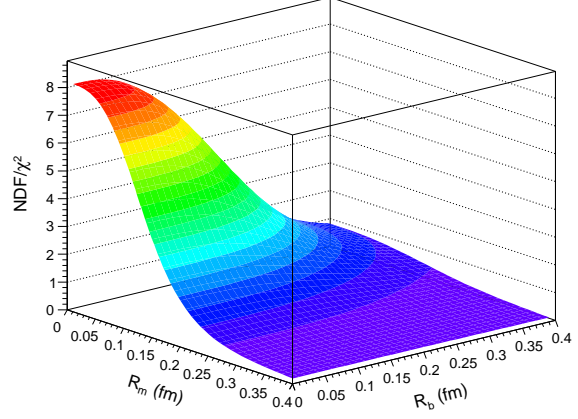


Figure 2: (Color online). Same as Fig. 1 but for LQCD data up to $T = 170$ MeV.

LQCD calculation. Figure 2 shows that NDF/χ^2 is maximum for $R_m = R_b = 0$. Hence the excluded volume effect can be ignored at the lower temperature ($T < T_c$). A similar comment was also there in the Ref. [22]. However, above this crossover temperature EVHRG model gives better descriptions of LQCD data compared to the ideal HRG model as shown in the Fig. 1.

Figure 3 shows the temperature dependence of normalised pressure and normalised energy density respectively at $\mu = 0$. Results of ideal HRG model have been represented by solid red lines. Other lines correspond to the results of the interacting HRG model or the EVHRG model with different combinations of radii of mesons and baryons. It can be seen that there is almost no effect of interaction till $T \simeq 120$ MeV both in pressure and energy density. The reason for this is that the effective degree of freedom of the system does not increase much up to this temperature and therefore correction due to excluded volume is small. Beyond $T = 120$ MeV a substantial change in these quantities has been observed. It can be seen from this figure that at large T normalised pressure as well as normalised energy density are suppressed compared to the ideal HRG if we take non-zero hardcore radii of the particles. This is expected since particles start interacting at large temperature where the hadronic population is large.

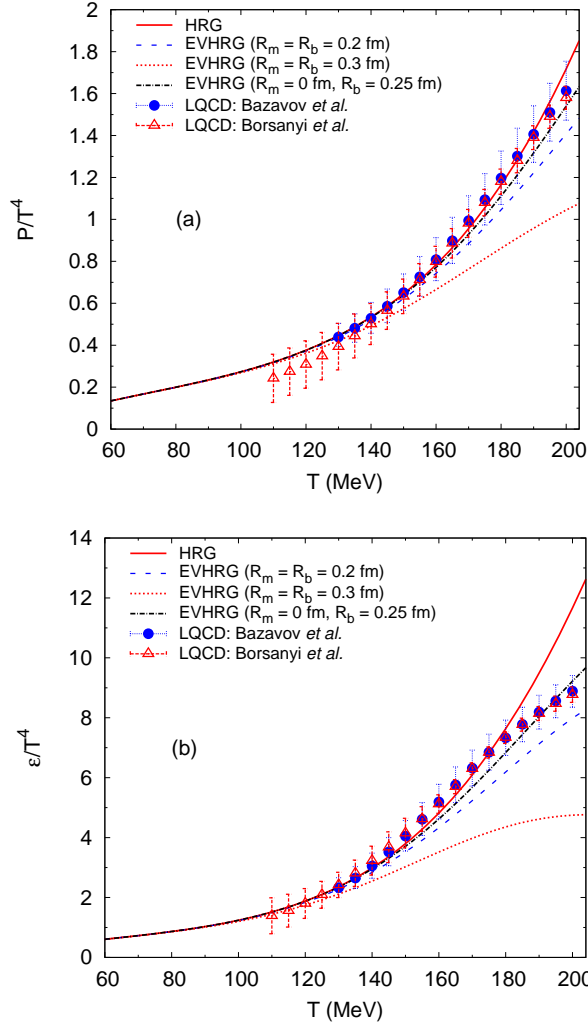


Figure 3: (Color online) Variation of normalised pressure and energy density with the temperature at $\mu = 0$. Continuum limit lattice QCD data are taken from Bazavov *et al.* [45] and Borsanyi *et al.* [46].

Further, suppression increases with the increase of radii of the particles. The continuum limit LQCD data of Bazavov *et al.* [45] and Borsanyi *et al.* [46] have also been plotted in this figure. Figure 3(a) illustrates the excellent agreement between the ideal HRG and the lattice QCD calculations of normalised pressure. On the other hand, one can see from Fig. 3(b) that normalised energy density calculated in ideal HRG model is close to the LQCD data up to $T \simeq T_c$. However, high temperature region agrees well with EVHRG model with $R_m = 0.0$ fm and $R_b = 0.25$ fm. It should be noted that this combination of R_m and R_b gives the best fit of the LQCD data up to $T = 200$ MeV which is already shown in the Fig. 1. The increase of the hardcore radii of all the particles further to 0.3 fm reduces the ability to reproduce both the LQCD results of normalised pressure and normalised energy density as can be seen from the Fig. 3.

4. Specific heat

The specific heat at constant volume C_V is given by [45]

$$C_V = \left(\frac{\partial \epsilon}{\partial T} \right)_V. \quad (13)$$

The C_V is a sensitive indicator of the transition from hadronic matter to the QGP. The C_V increases rapidly or even diverges near the transition temperature for a conventional second order phase transition.

Figure 4 shows temperature dependence of normalised specific heat at $\mu = 0$. Similar like normalised pressure and energy density, normalised specific heat calculated in ideal HRG model is very close to the continuum limit LQCD data [45, 46] up to the temperature $T \simeq T_c$. Results of normalised specific heat calculated in EVHRG model has also been shown in this figure. Normalised specific heat in EVHRG model is suppressed compared to ideal HRG model and the suppression increases with increasing temperature as well as with increasing radii of particles because of the repulsive interaction between hadrons.

5. P/T^4 and C_V/T^3 at chemical freeze-out

The thermal fireball created due to heavy ion collision expands and cools. At the initial stage, a large number of

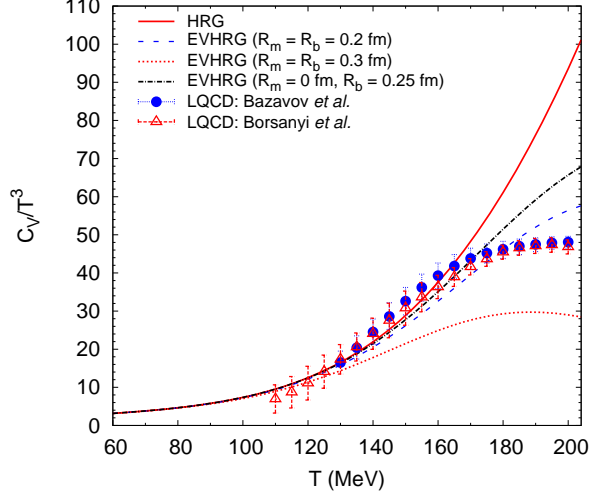


Figure 4: (Color online) Variation of normalised specific heat at constant volume with temperature at $\mu = 0$. Lattice QCD data for continuum extrapolation are taken from Bazavov *et al.* [45] and Borsanyi *et al.* [46].

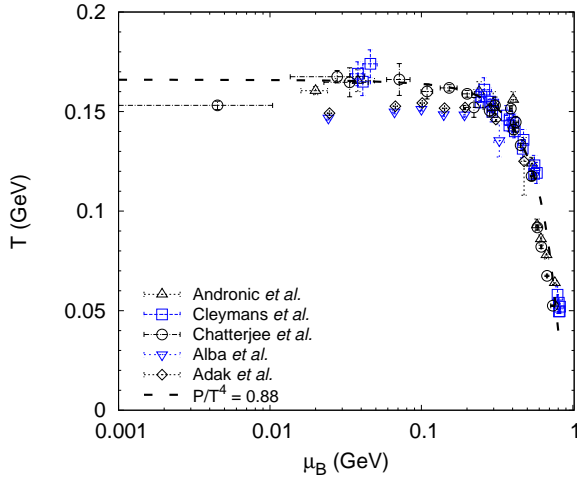


Figure 5: (Color online) Chemical freeze-out parameters (T, μ_B) obtained by different groups [16, 32, 35, 36, 41] at various $\sqrt{s_{NN}}$ along with the line of $P/T^4 = 0.88$ calculated in the ideal HRG model.

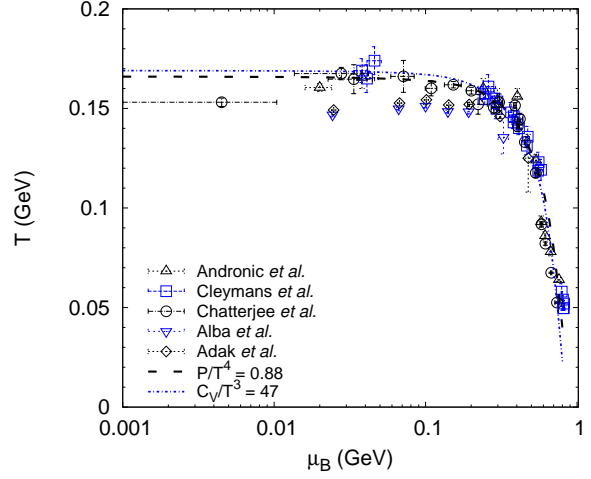


Figure 6: (Color online) Same as Fig. 5 but with an additional comparison with constant normalised specific heat.

particles are produced due to deposition of a huge amount of energy in the core of the collision. At this stage, particles collide mostly inelastically. After some time inelastic collisions among the particles stop and hence particle yields or ratios get fixed. This stage is called chemical freeze-out. It is already mentioned in the Sec. 1 that, from the experimental information about particle ratios or particle yields chemical freeze-out parameters can be calculated phenomenologically [15, 16, 35–39]. Although, all those calculations ignored any dynamics of the system. Chemical freeze-out parameters can also be calculated phenomenologically from experimental data of fluctuations [32, 41]. There are several universal chemical freeze-out descriptions in the existing literature which give quite satisfactory descriptions of the particle multiplicities measured in heavy-ion collisions. These universal chemical freeze-out properties include $\varepsilon/n = 1$ GeV [35, 47], $s/T^3 = 7$ [48], $n_B + n_{\bar{B}} = 0.12 \text{ fm}^{-3}$ [49], $(\varepsilon - 3P)/T^4 = 7/2$ [22] etc., where s is the entropy density, n_B is the baryon density and $n_{\bar{B}}$ is the anti-baryon density. In this paper, two more universal descriptions of chemical freeze-out have been proposed, namely $P/T^4 \simeq 0.88$ and $C_V/T^3 \simeq 47$. Figure 5 shows chemical freeze-out parameters in (T, μ_B) plane calculated in

HRG model by different groups [16, 32, 35, 36, 41] along with the line of $P/T^4 = 0.88$. Different symbols with error bars represent phenomenologically calculated chemical freeze-out parameters from experimental data at different $\sqrt{s_{NN}}$ ranging from a couple of GeV at SIS up to several TeV at LHC. The black dashed line in the Fig. 5 shows line of $P/T^4 = 0.88$ in (T, μ_B) plane calculated in the ideal HRG model in this present work. It can be seen that this constant normalised pressure can reproduce chemical freeze-out parameters at various $\sqrt{s_{NN}}$ in (T, μ_B) plane. Similarly, $C_V/T^3 = 47$ calculated in the ideal HRG model reproduces very well the chemical freeze-out diagram as can be seen from the Fig. 6. Further, it is observed that $\varepsilon/n = 1$ GeV, $s/T^4 = 6.4$, $n_B + n_{\bar{B}} = 0.1 \text{ fm}^{-3}$ and $(\varepsilon - 3P)/T^4 = 2.9$ etc. calculated in the ideal HRG model can also describe chemical freeze-out diagram. Hence this present work is consistent with the previous works related to universal chemical freeze-out descriptions [22, 35, 47–49]. It is worth mentioning that conservation laws given in Eqs. 6–7 have been incorporated during calculations of different observables in (T, μ_B) plane.

6. Conclusions

We conclude that below the crossover temperature, the ideal HRG model is good enough to describe LQCD data of pressure, energy density and specific heat calculated at $\mu = 0$. Further, the chemical freeze-out parameters deduced at various $\sqrt{s_{NN}}$ are well reproducible in the ideal HRG model using the conditions of constant normalised pressure and constant normalised specific heat respectively. It has been observed that both $P/T^4 = 0.88$ and $C_V/T^3 = 47$ calculated in ideal HRG model reproduce very well the chemical freeze-out diagram.

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